

General matter coupled $\mathcal{N} = 2$, $D = 5$ gauged supergravity

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Abstract

We give the full lagrangean and supersymmetry transformation rules for $D = 5$, $\mathcal{N} = 2$ supergravity interacting with an arbitrary number of vector, tensor and hyper-multiplets, with gauging of the R-symmetry group $SU(2)_R$ as well as a subgroup K of the isometries of the scalar manifold. Among the many possible applications, this theory provides the setting where a supersymmetric brane-world scenario could occur. We comment on the presence of AdS vacua and BPS solutions that would be relevant towards a supersymmetric smooth realization of the Randall-Sundrum “alternative to compactification”. We also add some remarks on the connection between this most general 5D fully coupled supergravity model and type IIB theory on the T^{11} manifold.

1 Introduction

In the quest for a unified description of gravity and matter interactions, several higher dimensional theories have been proposed in the past. In this respect, gauged supergravities, where the global isometries of the matter lagrangean are promoted to local symmetries, have been widely explored and by now almost all allowed models, for diverse spacetime dimensions and number \mathcal{N} of supersymmetries have been analysed (see for instance [1, 2]).

The five-dimensional $\mathcal{N} = 2$ supergravity theory, in particular, has been considered at various stages, but despite the many papers on the subject [3]–[11], it still lacks a complete description where all possible matter couplings are included and the most general gauging is performed.

The present renewed interest in gauged supergravity theories is mainly due to their prominent role within the *AdS/CFT* correspondence [12]. It is believed that the 5D gauged supergravities provide a consistent non-linear truncation of the lowest lying Kaluza–Klein modes of type IIB supergravity on an $AdS_5 \times X^5$ space, that should be dual to some four-dimensional supersymmetric conformal field theory. In detail, the truncation of the $AdS_5 \times S^5$ compactification [13] should be described by $\mathcal{N} = 8$ gauged supergravity [14], while the remaining $AdS_5 \times X^5$ models should correspond to $\mathcal{N} < 8$ gauged theories. A special attention is devoted to the $X^5 = T^{11}$ case [15], maybe the most tested non-trivial instance of the AdS/CFT correspondence [16, 17], whose low-energy action should be expressed in terms of an $\mathcal{N} = 2$ $U_R(1)$ gauged supergravity theory [17].

Still in the *AdS/CFT* framework, gauged supergravities are very interesting because they open the possibility of studying the renormalisation group (RG) flows of deformations of Yang–Mills theories by looking only at their supergravity formulation [18]–[21]. Supersymmetric [18] and non supersymmetric flows [19, 20] to other conformal or non-conformal theories have been studied, mostly using $\mathcal{N} = 8$ gauged supergravity.

In connection with the $\mathcal{N} = 1$ CFT dual to the $AdS_5 \times T^{11}$ compactification, the $\mathcal{N} = 2$ gauged theory could be useful to follow the RG flow associated with fractional branes [21], as well as a supersymmetric deformation which breaks the $SU(2) \times SU(2)$ flavour group to the diagonal $SU(2)$, as we will describe later.

Another line of development providing a strong motivation to find the most general $\mathcal{N} = 2$ gauged supergravity in $D = 5$ is the increasing phenomenological interest in brane-world scenarios¹ based on both heterotic M -theory compactifications [24, 9] and Randall–Sundrum type models [25, 26]. More precisely, one can distinguish between two setups. In the first (RS1) [25], meant to provide a solution to the hierarchy problem, there are two membranes located at the orbifold fixed points of a fifth compact dimension. The complete action includes 5D gravity and sources for the two membranes:

$$S = S_{bulk} + S_{branes}.$$

¹ The idea of embedding our universe into an uncompactified higher dimensional spacetime can be traced back to [22] and was further pursued in [23].

In the second (RS2) [26], there is a single membrane source where gravity is confined by a volcano potential given by two surrounding AdS spaces. Since in this case the fifth dimension is really uncompactified, RS2 suggests an alternative to Kaluza–Klein reduction, and one can aim at obtaining this model within a gravity theory, without singular sources. In order to realize RS2, one must find a model yielding two different stable critical points with equal values of the vacuum energy and a domain–wall solution interpolating between them.

Although it is perhaps desirable to embed any of the above scenarios into a supersymmetric string or gravity theory, the supersymmetrisation of the RS2 “alternative to compactification” is obviously much more appealing in view of its theoretical implications. Regrettably, Kallosh and Linde have shown [27] that none of the available $5D$ supergravity theories allow for such RS2 construction, and a definite answer for the minimal supersymmetric extension can only derive from the study of the fully coupled $5D$ $\mathcal{N} = 2$ theory. Notice that this is not in contrast with the result in [28], where the RS1 scenario is considered within the gauged $\mathcal{N} = 2$ pure supergravity [4] modified by the presence of singular sources.

Attempts to obtain any of these scenarios from a pure stringy perspective can be found in [29].

Aside from the supersymmetric brane worlds, the gauged and fully coupled $D = 5$ $\mathcal{N} = 2$ theory is the starting point for the study of five–dimensional black holes along the lines of [30].

The history of the couplings and gaugings of $D = 5$, $\mathcal{N} = 2$ supergravity is quite long. The pure theory was developed long ago [3], as its $U_R(1)$ –gauged version [4]. The interaction with vector multiplets, yielding the general Einstein–Maxwell ungauged theory, was proposed in [5], while some of its possible gaugings appeared in [6]. As a byproduct of the above–mentioned heterotic M –theory compactifications, hypermatter was later coupled to the abelian Einstein–Maxwell theory² [9].

Very recently, the addition of tensor multiplets obtained by dualising some of the vectors has been explored within the Einstein–Maxwell theory, with the gauging of a subgroup $K \subset G$ of the isometries of the scalar manifold and of a $U(1)$ subgroup of the R –symmetry group [10, 11].

This paper completes the above work by adding to the coupling of vector and tensor multiplets, also interaction with an arbitrary number of hyper–multiplets and generic gauging of $K \subset G$ and $SU(2)_R$. The scalar fields belonging to the vector and hyper–multiplets parametrise a manifold \mathcal{M} that is the product of a very special [32] by a quaternionic manifold. Rather than the Noether method, we use our past experience [33, 34] and construct the lagrangean by a geometrical technique that yields quite naturally

²The possibility of a coupling to hypermatter was foreseen in [31] in the context of M –theory compactifications over Calabi–Yau manifolds and a first description of such ungauged coupling can be found in [8].

also all the higher order terms in the fermion fields, that were often neglected in the past from both lagrangeans and supersymmetry transformation rules. In particular, the scalar potential of the theory can be completely expressed in terms of the shifts that appear in the supersymmetry rules of fermionic fields due to the gauging [35].

In sections 2–4, after a description of the generic matter couplings and $SU_R(2)$ and Yang–Mills gaugings, we write the general lagrangean and complete supersymmetry transformation rules. We find that the scalar field potential is modified by the inclusion of hypermultiplets in that a new term appears and the contribution due to the R -symmetry gauging is now given in terms of $SU(2)_R$ rather than $U(1)_R$ invariant objects.

We then turn in section 5 to the investigation of the possible realization of the RS2 scenario and perform a very simple preliminary analysis of the scalar potential along the lines of [27]. We seem to find that no new supersymmetric AdS vacua arise and no new BPS solutions are generated. However, the study of possible non-BPS solutions surely deserves a deep investigation that we postpone to future work.

As a further application of our results, we finish in section 6 with some remarks concerning the explicit realization of the theory corresponding to the T^{11} compactification of type IIB supergravity.

2 Preliminaries: Pure supergravity

Rather than component formalism and Noether method, we chose to work with superspace language for two main reasons: the first is the use of differential forms, that often simplify computations and make more transparent the geometric meaning of the various structures in the theory [2], the second is the use of superfields [36], that guarantee a natural way to achieve supersymmetry. The promotion of differential forms to superforms yields the supersymmetry transformations and equations of motion, without the need of an action to start from. The reduction to the ordinary component formalism is trivial and it shows that one obtains naturally all the supercovariantized quantities. Thus superspace formalism is also a good tool to simplify computations involving higher order fermion terms, which are often disregarded in other treatments.

In order to exemplify our technique and show how to analyze the results, we briefly revisit the pure $\mathcal{N} = 2$ five-dimensional supergravity [3, 4] in the superspace formalism [36].

The pure supergravity multiplet³

$$\{e_\mu^a, \psi_\mu^{\alpha i}, A_\mu\} \quad (2.1)$$

contains the graviton e_μ^a , two gravitini $\psi_\mu^{\alpha i}$ and a vector field A_μ (the graviphoton), that are described in superspace by the supervielbeins $e^a = (e^a, \psi^{\alpha i})$, the Lorentz connection ω_a^b and the one-form A .

³Our conventions are collected in the Appendix.

The torsion $T^{\underline{a}}$, the Lorentz and graviphoton curvatures $R_{\underline{a}}^{\underline{b}}$ and F are defined by

$$\begin{aligned} T^{\underline{a}} &= De^{\underline{a}} = de^{\underline{a}} + e^{\underline{b}}\omega_{\underline{b}}^{\underline{a}}, \\ R_{\underline{a}}^{\underline{b}} &= d\omega_{\underline{a}}^{\underline{b}} + \omega_{\underline{a}}^{\underline{c}}\omega_{\underline{c}}^{\underline{b}}, \\ F &= dA. \end{aligned} \tag{2.2}$$

These fields satisfy the Bianchi Identities (BI)

$$DT^{\underline{a}} = e^{\underline{b}}R_{\underline{b}}^{\underline{a}}, \quad DR_{\underline{a}}^{\underline{b}} = 0, \quad dF = 0. \tag{2.3}$$

It is well known that each tensor component represents a full superfield multiplet, containing a large number of component fields, most of which are superfluous. The unphysical fields are eliminated by imposing constraints on the supercurvatures.

Once the constraints are imposed, the BI of the various superfields are no longer automatically satisfied, and their consistent solution determines the couplings and the dynamics of the fields, through the derivation of the equations of motion.

Remarkably, not all the constraints are dynamical, but some of them can be absorbed in superfield redefinitions [37], that highly reduce the number of effective degrees of freedom and simplify the solution of the BI.

Without entering the technical details, we mention that the general strategy [37] based on group theoretical arguments leads to the fundamental constraint

$$T_{\alpha i \beta j}^a = \frac{1}{2}\epsilon_{ij}\Gamma_{\alpha\beta}^a, \tag{2.4}$$

which is needed to preserve rigid supersymmetry, and those imposing the dynamics ⁴:

$$T_{\alpha i \beta j}^{\sigma k} = 0, \tag{2.5}$$

$$F_{\alpha i \beta j} = -\frac{i}{4}\sqrt{6}\epsilon_{ij}C_{\alpha\beta}. \tag{2.6}$$

From the rheonomic approach point of view [2, 4], (2.4) and (2.6) are a result of the Maurer–Cartan’s equations dual to the $SU(2, 2|1)$ superalgebra.

One can now solve the BI and find the following parametrizations:

$$T^a = -\frac{1}{4}\bar{\psi}^i\gamma^a\psi_i, \tag{2.7a}$$

$$T^i = \frac{1}{2}e^ae^bT_{ba}{}^i + \frac{i}{4\sqrt{6}}e^a\left(\gamma_{abc}\psi^i - 4\eta_{ab}\gamma_c\psi^i\right)F^{bc}, \tag{2.7b}$$

$$\begin{aligned} R_{ab} &= \frac{1}{2}e^de^cR_{cdab} + \frac{1}{4}e^c\bar{\psi}^i\gamma_cT_{abi} - \frac{1}{2}e^c\bar{\psi}^i\gamma_{[a}T_{b]c}{}^i + \\ &+ \frac{i}{8\sqrt{6}}\left(\bar{\psi}_i\gamma_{ab}{}^{cd}\psi^i + 4\bar{\psi}_i\psi^i\eta_{ab}^{cd}\right)F^{cd}, \end{aligned} \tag{2.7c}$$

$$F = \frac{1}{2}e^ae^bF_{ba} + \frac{i\sqrt{6}}{8}\bar{\psi}^i\psi_i. \tag{2.7d}$$

⁴All the conventions have been chosen such that the supersymmetry laws and the structure of the Lagrangean match the formulae in [5]. Only the definition of the gravitational covariant derivative differs by a sign, that reflects in the opposite sign in the definitions of ω and R with respect to [5].

This solution deserves some comments. First of all, since the T - and F -BI are coupled, they cannot be solved separately, but the $T_{a\alpha i}{}^{\sigma k}$ component of the torsion is determined by the F -BI. Although the coupling with matter multiplets could in principle change the constraints (2.4)–(2.6) and their solution (2.7), one finds that the closure of the supersymmetry algebra still requires the fulfillment of the fundamental constraint (2.4). Moreover, it turns out that also the (2.7a) solution and the (2.5) constraint can be preserved. In particular, the $T^a{}_{bc}$ component of the torsion can always be annihilated by a shift of the Lorentz connection $\omega_{ab}{}^c$, translating all its non-zero components in the definition of $T_{a\alpha i}{}^{\beta j}$. E.g., in our formulation, the solution given in [4] would read

$$T_{ab}{}^c \sim \epsilon_{ab}{}^{cde} F_{de}$$

and

$$T_{a\alpha i}{}^{\beta j} \sim (\Gamma^b)_\alpha{}^\beta \delta_i^j F_{ab},$$

which differs from (2.7) by a shift of $\epsilon_{abcde} F^{de}$ in the $\omega_{ab}{}^c$ definition. For the (2.5) constraint, using the vielbein and connection redefinitions found in [37], one sees that only its $\underline{4}$ irreps. of $SO(5)$ are physical and they are fixed by the solution of the lowest dimensional T -BI which does not change in presence of matter⁵. The same field redefinitions also tell that the only physical component in $T_{a\alpha i}{}^b$ is the $\underline{40}$, which does not correspond to any structure in our supergravity model and indeed it is set to zero by the same T -BI.

Finally, the (2.7) equations can be determined by solving only the F - and T -BI, since by Dragon's theorem the R -BI follow once solved the T ones [37].

The ordinary supersymmetry transformations can be easily read off from the superspace results (2.7). In fact, $\varepsilon^A = (0, \varepsilon^{\alpha i})$ being the translation parameter, the supersymmetry transformations of the component fields are given by covariantized superspace Lie derivatives of the corresponding superfields, evaluated at $\vartheta = 0 = d\vartheta$:

$$\delta_\varepsilon \phi = (i_\varepsilon D + D i_\varepsilon) \phi|_{\vartheta=0=d\vartheta}. \quad (2.8)$$

They are explicitly

$$\begin{aligned} \delta_\varepsilon e^a &= \frac{1}{2} \bar{\varepsilon}^i \gamma^a \psi_i, \\ \delta_\varepsilon \psi^i &= D(\hat{\omega}) \varepsilon_i + \frac{i}{4\sqrt{6}} e^a (\gamma_{abc} - 4\eta_{ab} \gamma_c) \varepsilon_i \hat{F}^{bc}, \\ \delta_\varepsilon \omega_a{}^b &= \frac{i}{4\sqrt{6}} (\bar{\psi}_i \gamma_{ab}{}^{cd} \varepsilon^i + 4 \bar{\psi}_i \varepsilon^i \eta_{ab}^{cd}) \hat{F}^{cd}, \\ \delta_\varepsilon A &= \frac{i\sqrt{6}}{4} \bar{\psi}^i \varepsilon_i, \end{aligned} \quad (2.9)$$

where the hatted quantities refer to supercovariantized terms, i.e. $\hat{F}_{ab} = F_{ab} + \frac{i\sqrt{6}}{4} \bar{\psi}_{[a} \psi_{b]i}$.

⁵We will see in the next section that this freedom can be used to couple the hypermatter.

3 Gauged supergravity with generic matter coupling

Five-dimensional $\mathcal{N}=2$ supergravity allows, beside the supergravity multiplet (2.1), three kinds of matter multiplets: the vector, tensor and hypermultiplet.

The vector multiplet

$$\{A_\mu, \lambda^i, \phi\}$$

contains a vector field, an $SU_R(2)$ doublet of spin-1/2 fermions and one real scalar field, while the tensor multiplet

$$\{B_{\mu\nu}, \lambda^i, \phi\}$$

contains a tensor field of rank two, and again an $SU_R(2)$ doublet of spin-1/2 fermions and one real scalar field. In the hypermultiplet

$$\{\zeta^A, q^X\}$$

there is a doublet of spin-1/2 fermions $A = 1, 2$ and four real scalars $X = 1, \dots, 4$.

To allow for self-interactions, the scalars of the n_V vector, n_T tensors and n_H hypermultiplets parametrize a manifold \mathcal{M}_{scalar} which is the direct product of a very special [32] and a quaternionic manifold

$$\mathcal{M} = \mathcal{S}(n_V + n_T) \otimes \mathcal{Q}(n_H), \quad (3.1)$$

with $\dim_{\mathbb{R}} \mathcal{S} = n_V + n_T$ and $\dim_{\mathbb{Q}} \mathcal{Q} = n_H$.

In detail, the theory we are going to describe has the following field content

$$\{e_\mu^a, \psi_\mu^i, A_\mu^I, B_{\mu\nu}^M, \lambda^{i\tilde{a}}, \zeta^A, \phi^{\tilde{x}}, q^X\}. \quad (3.2)$$

Here $I = 0, 1, \dots, n_V$ is an index labeling the vector fields of the n_V vector multiplets and the graviphoton, since they will mix in the interactions. $M = 1, \dots, n_T$ labels the tensor multiplets. The scalars $\phi^{\tilde{x}}$, $\tilde{x} = 1, \dots, n_V + n_T$, parametrize the target space \mathcal{S} and thus \tilde{x} is a curved index. The $\lambda^{i\tilde{a}}$ instead transform as vectors under the tangent space group $SO(n_V + n_T)$ and $\tilde{a} = 1, \dots, n_V + n_T$ is the corresponding flat index. The \mathcal{Q} manifold is the target space of the q^X scalars and $X = 1, \dots, 4n_H$ are the curved indices labeling the coordinates. As expected for a quaternionic manifold, we have two types of flat indices $A = 1, \dots, 2n_H$ and $i = 1, 2$, corresponding to the fundamental representations of $USp(2n_H)$ and $USp(2) \simeq SU(2)$.

We will shortly see that it is also useful to introduce a collective index for the vector and tensor fields, which we denote by $\tilde{I} = (I, M)$.

3.1 The \mathcal{S} target space manifold

The scalar field target manifold of the vector and tensor multiplets \mathcal{S} is a very special manifold [5] which can be described by an $(n_V + n_T)$ -dimensional cubic hypersurface

$$C_{\tilde{I}\tilde{J}\tilde{K}} h^{\tilde{I}} h^{\tilde{J}} h^{\tilde{K}} = 1 \quad (3.3)$$

of an ambient space parametrized by $n_V + n_T + 1$ coordinates $h^{\tilde{I}} = h^{\tilde{I}}(\phi^{\tilde{x}})$. It is known that the kinetic term for such scalars can be written in terms of the coordinates as

$$C_{\tilde{I}\tilde{J}\tilde{K}} h^{\tilde{I}} \partial_\mu h^{\tilde{J}} \partial^\mu h^{\tilde{K}},$$

where $C_{\tilde{I}\tilde{J}\tilde{K}}$ is a completely symmetric *constant* tensor that determines also the Chern–Simons couplings of the vector fields.

A complete classification of the allowed homogeneous manifolds has been given in [32] and a lot of their interesting properties, especially when they are restricted to be a coset of the Jordan family, have been given in [5], to which we surely refer the reader for all the details.

Here we only collect some notions and results for their geometrical structures, which are directly related to the computations we present in this and the next sections.

First, we note that when $n_T \neq 0$ not all the $C_{\tilde{I}\tilde{J}\tilde{K}}$ coefficients differ from zero, but, as stated in [10], the only components that survive the gauging of a Yang–Mills group are the C_{IJK} and C_{IMN} .

For what concerns the \mathcal{S} manifold, $f_{\tilde{x}}^{\tilde{a}}$, $g_{\tilde{x}\tilde{y}}$ and $\Omega_{\tilde{x}}^{\tilde{a}\tilde{b}}$ denote its $(n_V + n_T)$ –bein, the metric and the spin connection, which can be given implicitly in terms of $f_{\tilde{x}}^{\tilde{a}}$ through the formula

$$f_{[\tilde{x},\tilde{y}]}^{\tilde{a}} + \Omega_{[\tilde{y}}^{\tilde{a}\tilde{b}} f_{\tilde{x}]}^{\tilde{b}} = 0.$$

The Riemann tensor is given by

$$K_{\tilde{x}\tilde{y}\tilde{z}\tilde{w}} = \frac{4}{3} \left(g_{\tilde{x}[\tilde{w}} g_{\tilde{z}]\tilde{y}} + T_{\tilde{x}[\tilde{w}}^{\tilde{t}} T_{\tilde{z}]\tilde{y}\tilde{t}} \right), \quad (3.4)$$

where $T_{\tilde{x}\tilde{y}\tilde{z}}$ is a completely symmetric function of $\phi^{\tilde{x}}$. The coordinates of the ambient space $h^{\tilde{I}}$ have an index which is raised and lowered through the $\phi^{\tilde{x}}$ –dependent metric $a_{\tilde{I}\tilde{J}}$.

All these functions are subject to the following algebraic and differential constraints, that are essential to close the supersymmetry algebra [5]:

$$\begin{aligned} C_{\tilde{I}\tilde{J}\tilde{K}} &= \frac{5}{2} h_{\tilde{I}} h_{\tilde{J}} h_{\tilde{K}} - \frac{3}{2} a_{(\tilde{I}\tilde{J}} h_{\tilde{K})} + T_{\tilde{x}\tilde{y}\tilde{z}} h_{\tilde{I}}^{\tilde{x}} h_{\tilde{J}}^{\tilde{y}} h_{\tilde{K}}^{\tilde{z}}, \\ h^{\tilde{I}} h_{\tilde{I}}^{\tilde{x}} &= 0, \quad h_{\tilde{x}}^{\tilde{I}} h_{\tilde{y}}^{\tilde{J}} a_{\tilde{I}\tilde{J}} = g_{\tilde{x}\tilde{y}}, \\ a_{\tilde{I}\tilde{J}} &= h_{\tilde{I}} h_{\tilde{J}} + h_{\tilde{I}}^{\tilde{x}} h_{\tilde{J}}^{\tilde{y}} g_{\tilde{x}\tilde{y}}, \\ h_{\tilde{I},\tilde{x}} &= \sqrt{\frac{2}{3}} h_{\tilde{I}\tilde{x}}, \quad h^{\tilde{I},\tilde{x}} = -\sqrt{\frac{2}{3}} h_{\tilde{x}}^{\tilde{I}}, \\ h_{\tilde{I}\tilde{x};\tilde{y}} &= \sqrt{\frac{2}{3}} \left(g_{\tilde{x}\tilde{y}} h_{\tilde{I}} + T_{\tilde{x}\tilde{y}\tilde{z}} h_{\tilde{I}}^{\tilde{z}} \right), \quad h^{\tilde{I}}_{\tilde{x};\tilde{y}} = -\sqrt{\frac{2}{3}} \left(g_{\tilde{x}\tilde{y}} h^{\tilde{I}} + T_{\tilde{x}\tilde{y}\tilde{z}} h^{\tilde{z}\tilde{I}} \right). \end{aligned} \quad (3.5)$$

3.2 The \mathcal{Q} target manifold

Self-interacting hypermultiplets in an $\mathcal{N} = 2$, $D = 5$ theory are known to live on a quaternionic Kähler manifold [38, 8, 9]. The quaternionic metric tensor will be denoted by $g_{XY}(q)$, while $\omega_X{}^j{}_\iota(q)$ and $\omega_X{}^B{}_A(q)$ will be the $USp(2)$ and $USp(2n_H)$ connections.

As a consequence of its quaternionic structure, the holonomy group of the manifold \mathcal{Q} is a direct product of $SU(2)$ and some subgroup of the symplectic group in $2n_H$ dimensions. This means that one can introduce the vielbeins f_{iA}^X to pass to the flat indices $iA \in USp(2) \otimes USp(2n_H)$.

These vielbeins obey the following relations:

$$\begin{aligned} g_{XY} f_{iA}^X f_{jB}^Y &= \epsilon_{ij} C_{AB}, \\ f_{iC}^X f_j^{YC} + f_{iC}^Y f_j^{XC} &= g^{XY} \epsilon_{ij}, \\ f_{iA}^X f_B^{Yi} + f_{iA}^Y f_B^{Xi} &= \frac{1}{n_H} g^{XY} C_{AB}, \end{aligned} \tag{3.6}$$

where ϵ_{ij} and C_{AB} are the $SU(2)$ and $USp(2n_H)$ invariant tensors respectively.

To obtain the coupling of these fields to supergravity when they are chargeless with respect to the Yang–Mills (Maxwell) fields, one uses the freedom left by the solution of the T –BI of introducing some fermions in $T_{\alpha i \beta j}^{\gamma k}$, as explained in the analogous six–dimensional case [33].

Since the structure of the possible spinor component $T_{\alpha i \beta j}^{\gamma k}$ is of the same form as the pullback of the $USp(2)$ connection on the cotangent bundle basis of the superspace, one can write the new constraint

$$T_{\alpha i \beta j}^{\gamma k} = \delta_{\alpha}^{\gamma} \omega_{\beta j i}^k + (\alpha i \leftrightarrow \beta j), \tag{3.7}$$

and solve the BI, but this of course breaks $USp(2)$ covariance. It is more convenient to proceed in a slightly different but equivalent way which does not break covariance.

Redefining the covariant derivative D by introducing the $USp(2) \otimes USp(2n_H)$ connections

$$D = d + \omega_{Lorentz} + dq^X \omega_{X\#}^{\#}, \quad \# = \{A, i\}, \tag{3.8}$$

one gets the new torsion definitions

$$\begin{aligned} T^a &\equiv de^a + e^b \omega_b^a, \\ T^{\alpha i} &\equiv d\psi^{\alpha i} + \psi^{\beta i} \omega_{\beta}^{\alpha} + \psi^{\alpha j} \omega_j^i, \end{aligned} \tag{3.9}$$

and imposes again the constraint $T_{\alpha i \beta j}^{\gamma k} = 0$. This modifies also the T –BI according to

$$D_{(new)} T^{\alpha i} = \psi^{\beta i} R_{\beta}^{\alpha} + \psi^{\alpha j} \mathcal{R}_j^i, \tag{3.10}$$

where $\mathcal{R}_j^i \equiv d\omega_j^i + \omega_j^k \omega_k^i = \frac{1}{2} dq^X dq^Y \mathcal{R}_{XYj}^i$ is the $USp(2)$ curvature.

It is known [38] that a quaternionic manifold is maximally symmetric. This fixes the $USp(2)$ curvature to

$$\mathcal{R}_{XYij} = \kappa (f_{XiC} f_{Yj}^C - f_{YiC} f_{Xj}^C), \tag{3.11}$$

with κ a constant fixed by supersymmetry requirements to $\kappa = -1$.

Other useful identities regarding the definition of the \mathcal{Q} -Riemann tensor and the $USp(2n_H)$ curvature are

$$\mathcal{R}_{XYAB} = \kappa \left(f_{XiA} f_{YB}^i - f_{YiA} f_{XB}^i \right) + f_X^{iC} f_{Yi}^D \Omega_{ABCD}, \quad (3.12)$$

$$\mathcal{R}_{XYWZ} f_{iA}^W f_{jB}^Z = \epsilon_{ij} \mathcal{R}_{XYAB} + C_{AB} \mathcal{R}_{XYij}, \quad (3.13)$$

that can easily be pulled back on superspace. Here Ω_{ABCD} denotes the totally symmetric tensor of $USp(2n_H)$.

3.3 The Gauging

The gauging of matter coupled $\mathcal{N} = 2$ supergravity theories is achieved by identifying the gauge group K as a subgroup of the isometries G of the \mathcal{M} product space. If one choses to gauge $n_V + 1$ vector fields, one is left with up to $n_T = \dim G - n_V$ other ones, charged under K , which will be dualised to tensor fields. As explained in [34], two main cases can occur: K non abelian and $K = U(1)^{n_V+1}$. In the first case, supersymmetry requires K to be a subgroup of the full \mathcal{M} , and the hypermultiplet space will generically split into

$$n_H = \sum_i n_i R_i + \frac{1}{2} \sum_l n_l^P R_l^P,$$

where R_i and R_l^P are a set of irreps of K ($P = \text{pseudoreal}$). In the abelian case, the \mathcal{S} -manifold is not required to have any isometry and if the hypermultiplets are charged with respect to the $n_V + 1$ $U(1)$'s, the \mathcal{Q} manifold should at least have $n_V + 1$ abelian isometries.

The gauging now proceeds by introducing $n_V + 1$ Killing vectors acting generically on \mathcal{M} :

$$\begin{aligned} \phi^{\tilde{x}} &\rightarrow \phi^{\tilde{x}} + \epsilon^I K_I^{\tilde{x}}(\phi), \\ q^X &\rightarrow q^X + \epsilon^I K_I^X(q), \end{aligned}$$

for an infinitesimal parameter ϵ^I .

The quaternionic structure of \mathcal{Q} implies that K_I^X can be determined in terms of the Killing prepotential $P_{Ii}{}^j(q)$ [34], which satisfies

$$\mathcal{D}q^Y K_I^X \mathcal{R}_{XYi}{}^j = \mathcal{D}P_{Ii}{}^j \quad (3.14)$$

and, for f_{IJ}^K the gauge group structure constants,

$$g \mathcal{R}_{XYi}{}^j K_I^X K_J^Y + g_R P_{[Ii}{}^k P_{J]k}{}^j + g \frac{1}{2} f_{IJ}^K P_{Ki}{}^j = 0. \quad (3.15)$$

Gauging the supergravity theory is now done by gauging the composite connections of the underlying σ -model. Following the well-established general procedure [1, 2] first introduced in [39] for the $\mathcal{N} = 8$, $D = 4$ case, we proceed by replacing for the YM

couplings the covariant derivatives on the scalar and fermion fields containing the Lorentz, $SO(n_V + n_T)$ and $USp(2) \otimes USp(2n_H)$ connections by K -covariant derivatives

$$\begin{aligned}\mathcal{D}\phi^{\tilde{x}} &= D\phi^{\tilde{x}} + gA^I K_I^{\tilde{x}}(\phi), \\ \mathcal{D}q^X &= Dq^X + gA^I K_I^X(q), \\ \mathcal{D}\lambda_i^{\tilde{a}} &= D\lambda_i^{\tilde{a}} + gA^I L_I^{\tilde{a}\tilde{b}}(\phi)\lambda_i^{\tilde{b}}, \\ \mathcal{D}\zeta^A &= D\zeta^A + gA^I \omega_{IB}^A(q)\zeta^B,\end{aligned}\tag{3.16}$$

where g is the coupling constant, A^I the gauge field one-forms, $L_I^{\tilde{a}\tilde{b}}$ the G -transformation matrices of the gluinos

$$L_I^{\tilde{a}\tilde{b}} \equiv \partial^{\tilde{b}} K_I^{\tilde{a}}\tag{3.17}$$

and

$$\omega_{IB}^A \equiv K_{IX;Y} f_i^{XA} f_B^{Yi}.\tag{3.18}$$

At the same time, the connection $\Omega_{\tilde{a}\tilde{b}}$ is replaced by its gauged counterpart

$$\mathcal{D}\phi^{\tilde{x}}\Omega_{\tilde{x}\tilde{a}\tilde{b}} + gA^I K_{I\tilde{a};\tilde{b}}.\tag{3.19}$$

This reflects into suitable changes in the definition of the gauged curvatures and BI:

$$\mathcal{D}^2\phi^{\tilde{x}} = gF^I K_I^{\tilde{x}},\tag{3.20}$$

$$\mathcal{D}^2q^X = gF^I K_I^X,\tag{3.21}$$

$$\mathcal{D}^2\lambda^{\tilde{a}i} = \mathcal{R}_j^i \lambda^{\tilde{a}j} + K^{\tilde{a}\tilde{b}} \lambda^{\tilde{b}i} + gF^I L_I^{\tilde{a}\tilde{b}} \lambda^{\tilde{b}i},\tag{3.22}$$

$$\mathcal{D}^2\zeta^A = \mathcal{R}_B^A \zeta^B + gF^I \omega_{IB}^A \zeta^B,\tag{3.23}$$

where

$$K_{\tilde{a}\tilde{b}} = \frac{1}{2} \mathcal{D}\phi^{\tilde{x}} \mathcal{D}\phi^{\tilde{y}} K_{\tilde{y}\tilde{x}\tilde{a}\tilde{b}},\tag{3.24}$$

$$\mathcal{R}_{AB} = \frac{1}{2} \mathcal{D}q^X \mathcal{D}q^Y \mathcal{R}_{YXAB},\tag{3.25}$$

are the \mathcal{S} Riemann and $USp(2n_H)$ curvatures, with components defined in (3.4), (3.12) and

$$F^I = dA^I - \frac{1}{2} g f_{JK}^I A^J A^K\tag{3.26}$$

is the gauge field strength satisfying the BI

$$\mathcal{D}F^I = 0.\tag{3.27}$$

For the $SU_R(2)$ connection, the existence of a Killing vector prepotential $P_{Ii}^j(q)$ allows the following definition:

$$\omega_i^j \rightarrow \omega_i^j + g_R A^I P_{Ii}^j(q),\tag{3.28}$$

which replaces the $SU(2)_R$ connection with its gauged counterpart. This implies that the new covariant derivative acting on the gravitino is

$$T^{\alpha i} \equiv \mathcal{D}\psi^{\alpha i} = d\psi^{\alpha i} + \psi^{\beta i}\omega_{\beta}^{\alpha} + \psi^{\alpha j}\omega_j^i + g_R \psi^{\alpha j} A^I P_{Ij}^i(q), \quad (3.29)$$

and that the $SU_R(2)$ curvature definition is replaced by

$$\widehat{\mathcal{R}}_i^j = \frac{1}{2} \mathcal{D}q^X \mathcal{D}q^Y \mathcal{R}_{YX}^j + g_R F^I P_{Ii}^j, \quad (3.30)$$

provided P_{Ii}^j satisfies the (3.14) and (3.15) relations.

This leads to a further redefinition of the gaugino BI (3.22) and of the torsion one, which now read

$$\mathcal{D}^2 \lambda^{\tilde{a}i} = \widehat{\mathcal{R}}_j^i \lambda^{\tilde{a}j} + K^{\tilde{a}\tilde{b}} \lambda^{\tilde{b}i} + g F^I L_I^{\tilde{a}\tilde{b}} \lambda^{\tilde{b}i} \quad (3.31)$$

and

$$\mathcal{D}T^{\alpha i} = \psi^{\beta i} R_{\beta}^{\alpha} + \psi^{\alpha j} \widehat{\mathcal{R}}_j^i. \quad (3.32)$$

3.4 The solution of the Bianchi Identities

In order to solve the superspace BI in presence of gauging and to obtain the new susy rules, one must first face a technical problem, that is how to implement the tensor multiplet BI in superspace.

It is known [10] that the $B_{\mu\nu}^M$ fields satisfy a first order equation of motion of the type

$$\mathcal{D}B^M = \mathcal{M}^M_N \star B^N, \quad (3.33)$$

where \mathcal{M} is a mass matrix. The problem is that (3.33) contains the Hodge star product, which is not well defined in superspace. Moreover, being a first order equation of motion, (3.33) cannot be derived by the standard procedure of solving the BI for the superfield two-form B^M , after imposing some constraint on its field strength H^M .

It must also be noted that the B^M transform under the gauge group K and the correct definition of H^M is [10]

$$H^M \equiv dB^M + g \Lambda_{IN}^M A^I B^N, \quad (3.34)$$

where Λ_{IN}^M is the representation matrix⁶.

This implies that the H -BI become

$$\mathcal{D}H^M = g \Lambda_{IN}^M F^I B^N \quad (3.35)$$

where the superfield connection B^M appears explicitly, breaking covariance.

The solution to this problem lies in the origin of the tensor fields.

In perfect analogy with the six-dimensional case [40], before the gauging the B^M degrees of freedom are described by A^M vectors, transforming nontrivially under K . To

⁶It has been shown [10] that they must lie in a symplectic representation of the gauge group K .

obtain the gauging of K , one must also introduce some b^M tensor fields with the usual invariance $\delta b^M = d\Lambda^M$. These must be introduced in the lagrangean and transformation rules in such a way that everywhere F^M gets replaced by the combination

$$B^M = b^M + F^M. \quad (3.36)$$

Closure of the supersymmetry algebra imposes that the vector field A^M transforms non-trivially under the gauge transformations of the tensor fields

$$\delta b^M = d\Lambda^M \Rightarrow \delta A^M = -\Lambda^M, \quad (3.37)$$

leaving the (3.36) combination gauge invariant.

This allows the vector fields to be completely gauged away by taking $\Lambda^M = A^M$, leaving $B^M = b^M$, where now b^M has “eaten” the A^M degrees of freedom, and obtained its longitudinal modes by a Higgs-type mechanism. This also implies that B^M is now massive, and its supersymmetry variation acquires an additional term of the form $d\delta_{susy}A^M$.

From the superspace point of view, this can be seen as the need of imposing directly on B^M the same constraints and F -BI solutions imposed on F^M (which is now allowed, since the massive B^M has lost the gauge invariance under $\delta B^M = d\Lambda$) and then solve consistently the H -BI. These BI now will also provide the first-order B^M equations of motion at the level of the $ab\alpha i\beta j$ sector.

Following [10], from now on we will use $\mathcal{H}^{\tilde{I}}$ to denote collectively the F^I and B^M fields, depending on the value of \tilde{I} .

The constraints to be imposed on the curvatures are the straightforward generalization of those proposed for pure supergravity (2.4)–(2.6), and read

$$T_{\alpha i\beta j}^a = \frac{1}{2}\epsilon_{ij}\Gamma_{\alpha\beta}^a, \quad (3.38a)$$

$$\mathcal{H}_{\alpha i\beta j}^{\tilde{I}} = -\frac{\sqrt{6}}{4}i\epsilon_{ij}C_{\alpha\beta}h^{\tilde{I}}, \quad (3.38b)$$

$$T_{\alpha i\beta j}^{\gamma k} = 0. \quad (3.38c)$$

In addition, one must fix the normalization of the fermion fields:

$$\mathcal{D}_{\alpha i}\phi^{\tilde{x}} \equiv -\frac{i}{2}f_{\tilde{a}}^{\tilde{x}}\lambda_{\alpha i}^{\tilde{a}}, \quad (3.39)$$

$$\mathcal{D}_{\alpha i}q^X \equiv if_{Ai}^X\zeta_{\alpha}^A. \quad (3.40)$$

Introducing the (3.38) constraints and the (3.39)–(3.40) definitions in the BI, we obtain the new parametrizations of the curvatures as well as some algebraic and differential constraints on the geometric structures (e.g. those presented in (3.5)) and the equations of motion.

The torsion parametrization remains the same as in the pure gravity case:

$$T^a = -\frac{1}{4}\overline{\psi}^i\gamma^a\psi_i. \quad (3.41)$$

However, the super-field-strength of the gravitino now contains many new terms involving couplings with fermions and a new gauging term, which is fixed by the solution of the T -BI:

$$\begin{aligned}
T_i &= \frac{1}{2}e^a e^b T_{ba i} + \frac{i}{4\sqrt{6}}h_{\tilde{I}}e^a(\gamma_{abc}\psi_i - 4\eta_{ab}\gamma_c\psi_i)\mathcal{H}^{bc\tilde{I}} + \\
&- \frac{1}{12}e^a\gamma_{ab}\psi^j\bar{\lambda}_i\gamma^b\lambda_j^{\tilde{c}} + \frac{1}{48}e^a\gamma_{abc}\psi^j\bar{\lambda}_i\gamma^{bc}\lambda_j^{\tilde{a}} + \frac{1}{6}e^a\psi^j\bar{\lambda}_i\gamma_a\lambda_j^{\tilde{c}} + \\
&- \frac{1}{12}e^a\gamma^b\psi^j\bar{\lambda}_i\gamma_{ab}\lambda_j^{\tilde{c}} + \frac{1}{8}e^a\gamma^{bc}\psi_i\bar{\zeta}_A\gamma_{abc}\zeta^A + \frac{i}{\sqrt{6}}g_R e^a\gamma_a\psi^j P_{ij}, \tag{3.42}
\end{aligned}$$

where

$$P_{ij} \equiv h^I P_{Iij}. \tag{3.43}$$

Regarding the Yang-Mills and tensor multiplets, the scalar parametrization directly follows from (3.39):

$$\mathcal{D}\phi^{\tilde{x}} = e^a \mathcal{D}_a \phi^{\tilde{x}} + \frac{i}{2}\bar{\psi}^i \lambda_i^{\tilde{a}} f_{\tilde{a}}^{\tilde{x}}. \tag{3.44}$$

The F^I field-strength, apart from the component fixed in (3.38b), has a new term involving the gluino fields which is fixed by the F -BI. Since the B^M parametrization must have the same form, these can be collectively written as

$$\mathcal{H}^{\tilde{I}} = \frac{1}{2}e^a e^b \mathcal{H}_{ba}^{\tilde{I}} - \frac{1}{2}e^a \bar{\psi}^i \gamma_a \lambda_i^{\tilde{a}} h_{\tilde{a}}^{\tilde{I}} + \frac{i\sqrt{6}}{8}\bar{\psi}^i \psi_i h^{\tilde{I}}. \tag{3.45}$$

The gluinos field strength (and supersymmetry variation) are

$$\begin{aligned}
\mathcal{D}\lambda_i^{\tilde{a}} &= e^a \mathcal{D}_a \lambda_i^{\tilde{a}} - \frac{i}{2}f_{\tilde{x}}^{\tilde{a}}\gamma^a\psi_i \mathcal{D}_a \phi^{\tilde{x}} + \frac{1}{4}h_{\tilde{I}}^{\tilde{a}}\gamma^{ab}\psi_i \mathcal{H}_{ab}^{\tilde{I}} + \\
&- \frac{i}{4\sqrt{6}}T^{\tilde{a}}_{\tilde{b}\tilde{c}}\left[-3\psi^j\left(\bar{\lambda}_i^{\tilde{b}}\lambda_j^{\tilde{c}}\right) + \gamma_a\psi^j\left(\bar{\lambda}_i^{\tilde{b}}\gamma^a\lambda_j^{\tilde{c}}\right) + \frac{1}{2}\gamma_{ab}\psi^j\left(\bar{\lambda}_i^{\tilde{b}}\gamma^{ab}\lambda_j^{\tilde{c}}\right)\right] + \\
&+ g_R\psi^j P_{ij}^{\tilde{a}} + gW^{\tilde{a}}\psi_i, \tag{3.46}
\end{aligned}$$

where

$$P_{ij}^{\tilde{a}} \equiv h^{\tilde{a}I} P_{Iij} \tag{3.47}$$

and

$$W^{\tilde{a}} \equiv -\frac{\sqrt{6}}{8}\Omega^{MN}h_M^{\tilde{a}}h_N. \tag{3.48}$$

The (3.46) parametrization contains the obvious terms needed to close the F -BI and the supersymmetry algebra on $\phi^{\tilde{x}}$. In addition, the first bilinear in the gluini and the R -symmetry gauging terms are fixed by closure of the supersymmetry algebra. The Yang-Mills gauging term is then fixed by the H -BI, since it appears only when tensor multiplets are involved [10].

We point out that, differently from the four-dimensional case, the Yang-Mills gauging in the absence of tensor multiplets does not lead to any extra term. This is due to

the fact that, according to [6], $h^I K_I^{\tilde{x}} = 0$ if we assume that under an infinitesimal K -transformation with parameter η , the vectors transform as

$$\delta_\eta A^I \sim f_{JK}^I A^J \eta^K. \quad (3.49)$$

When the tensor fields are involved, the Killing vectors get a new contribution coming from the K -transformation properties of the tensors and therefore now $h^I K_I^{\tilde{x}} \neq 0$. This gives us back a Yang–Mills gauging term, determined by closure of the supersymmetry algebra on $\phi^{\tilde{x}}$, which must be equivalent to (3.48) and it reads

$$W^{\tilde{a}} = \frac{\sqrt{6}}{4} h^I K_I^{\tilde{x}} f_{\tilde{x}}^{\tilde{a}}. \quad (3.50)$$

The solution of the H -BI fixes the H^M parametrization, which reads

$$H^M = \frac{1}{3!} e^a e^b e^c H_{cba} + \frac{i}{8} g e^b e^a \bar{\psi}^i \gamma_{ab} \lambda_i^{\tilde{a}} h_N \Omega^{MN} + g \frac{\sqrt{6}}{16} e^a \bar{\psi}^i \gamma_a \psi_i \Omega^{MN} h_N, \quad (3.51)$$

where it must be noted that in H_{abc} one should substitute the B equations of motion.

The closure of the H -BI also imposes some constraints on the representation matrix Λ :

$$\begin{aligned} \Omega^{MN} h_N &= \sqrt{6} \Lambda_{IN}^M h^I h^N, \\ \Omega^{MN} h_N^{\tilde{a}} &= \sqrt{6} \Lambda_{IN}^M (h^{\tilde{a}I} h^N + h^I h^{\tilde{a}N}), \end{aligned} \quad (3.52)$$

which can be shown to be equivalent to condition (5.12) of [10], namely

$$\Lambda_{IN}^M = \frac{2}{\sqrt{6}} \Omega^{MP} C_{NPI}. \quad (3.53)$$

Turning to the hypermultiplets, the scalar parametrization is fixed once (3.40) is imposed, and is given by

$$\mathcal{D}q^X = e^a \mathcal{D}_a q^X - i \bar{\psi}^i \zeta^A f_{iA}^X. \quad (3.54)$$

Closure of supersymmetry on q^X and ζ^A imposes then

$$\mathcal{D}\zeta^A = e^a \mathcal{D}_a \zeta^A - \frac{i}{2} \gamma_a \psi^i \mathcal{D}_a q^X f_{iX}^A + g \psi^i \mathcal{N}_i^A, \quad (3.55)$$

with the g -order shift defined as

$$\mathcal{N}_{iA} \equiv \frac{\sqrt{6}}{4} f_{XAi} K_I^X h^I. \quad (3.56)$$

This term is due to the Yang–Mills charge of the hypermultiplets.

Finally, for completeness, we give here also the parametrization of the Riemann tensor:

$$\begin{aligned} R_{ab} &= \frac{1}{2} e^d e^c R_{cd,ab} + \frac{i}{8\sqrt{6}} (\bar{\psi}_i \gamma_{ab}{}^{cd} \psi^i + 4 \bar{\psi}_i \psi^i \eta_{ab}^{cd}) \mathcal{H}_{cd}^{\tilde{I}} h_{\tilde{I}} + \\ &+ \frac{1}{24} \bar{\psi}^i \gamma_{abc} \psi^j \bar{\lambda}_i \gamma^c \lambda_j + \frac{1}{48} \bar{\psi}^i \gamma_c \psi^j \bar{\lambda}_i \gamma^{abc} \lambda_j + \\ &+ \frac{1}{24} \bar{\psi}^i \psi^j \bar{\lambda}_i \gamma_{ab} \lambda_j + \frac{1}{4} \bar{\psi}_i \gamma^c \psi_j \zeta_A \gamma_{abc} \zeta^A - \frac{i}{2\sqrt{6}} \bar{\psi}^i \gamma_{ab} \psi^j P_{ij} + \\ &+ \frac{1}{4} e^c \bar{\psi}^i \gamma_c T_{abi} - \frac{1}{2} e^c \bar{\psi}^i \gamma_{[a} T_{b]ci}. \end{aligned} \quad (3.57)$$

As shown in the previous section, the ordinary supersymmetry transformations of the fields are recovered from the above superspace results by using (2.8). One sees that, as expected, they complete those given in [10] with the higher order Fermi terms and the new couplings:

$$\delta_\varepsilon e^a = \frac{1}{2} \bar{\varepsilon}^i \gamma^a \psi_i, \quad (3.58)$$

$$\begin{aligned} \delta_\varepsilon \psi_i &= \mathcal{D}(\hat{\omega}) \varepsilon_i + \frac{i}{4\sqrt{6}} h_{\tilde{I}} e^a (\gamma_{abc} \varepsilon_i - 4\eta_{ab} \gamma_c \varepsilon_i) \widehat{\mathcal{H}}^{bc\tilde{I}} - \delta_\varepsilon q^X \omega_{X i}{}^j \psi_j + \\ &- \frac{1}{12} e^a \gamma_{ab} \varepsilon^j \bar{\lambda}_i^{\tilde{c}} \gamma^b \lambda_j^{\tilde{c}} + \frac{1}{48} e^a \gamma_{abc} \varepsilon^j \bar{\lambda}_i^{\tilde{a}} \gamma^{bc} \lambda_j^{\tilde{a}} + \frac{1}{6} e^a \varepsilon^j \bar{\lambda}_i^{\tilde{c}} \gamma_a \lambda_j^{\tilde{c}} + \\ &- \frac{1}{12} e^a \gamma^b \varepsilon^j \bar{\lambda}_i^{\tilde{c}} \gamma_{ab} \lambda_j^{\tilde{c}} + \frac{1}{8} e^a \gamma^{bc} \varepsilon_i \bar{\zeta}_A \gamma_{abc} \zeta^A + \frac{i}{\sqrt{6}} g_R e^a \gamma_a \varepsilon^j P_{ij}, \end{aligned} \quad (3.59)$$

$$\delta_\varepsilon \phi^{\tilde{x}} = \frac{i}{2} \bar{\varepsilon}^i \lambda_i^{\tilde{a}} f_{\tilde{a}}^{\tilde{x}}, \quad (3.60)$$

$$\delta_\varepsilon A^I = \vartheta^I, \text{ where } \vartheta^{\tilde{I}} \equiv -\frac{1}{2} e^a \bar{\varepsilon}^i \gamma_a \lambda_i^{\tilde{a}} h_{\tilde{a}}^{\tilde{I}} + \frac{i\sqrt{6}}{4} \bar{\psi}^i \varepsilon_i h^{\tilde{I}}, \quad (3.61)$$

$$\begin{aligned} \delta_\varepsilon \lambda_i^{\tilde{a}} &= -\frac{i}{2} f_{\tilde{x}}^{\tilde{a}} \gamma^a \varepsilon_i \widehat{\mathcal{D}}_a \phi^{\tilde{x}} - \delta_\varepsilon \phi^{\tilde{x}} \Omega_{\tilde{x}}^{\tilde{a}\tilde{b}} \lambda_i^{\tilde{b}} - \delta_\varepsilon q^X \omega_{X i}{}^j \lambda_j^{\tilde{a}} + \frac{1}{4} h_{\tilde{I}}^{\tilde{a}} \gamma^{ab} \varepsilon_i \widehat{\mathcal{H}}_{ab}^{\tilde{I}} + \\ &- \frac{i}{4\sqrt{6}} T_{\tilde{b}\tilde{c}}^{\tilde{a}} \left[-3\varepsilon^j \bar{\lambda}_i^{\tilde{b}} \lambda_j^{\tilde{c}} + \gamma_a \varepsilon^j \bar{\lambda}_i^{\tilde{b}} \gamma^a \lambda_j^{\tilde{c}} + \frac{1}{2} \gamma_{ab} \varepsilon^j \bar{\lambda}_i^{\tilde{b}} \gamma^{ab} \lambda_j^{\tilde{c}} \right] + \\ &+ g_R \varepsilon^j P_{ij}^{\tilde{a}} + g W^{\tilde{a}} \varepsilon_i, \end{aligned} \quad (3.62)$$

$$\delta_\varepsilon B^M = d\vartheta^M + \frac{i}{8} g e^b e^a \bar{\varepsilon}^i \gamma_{ab} \lambda_i^{\tilde{a}} h_N \Omega^{MN} + g \frac{\sqrt{6}}{8} e^a \bar{\psi}^i \gamma_a \varepsilon_i \Omega^{MN} h_N, \quad (3.63)$$

$$\delta_\varepsilon q^X = -i \bar{\varepsilon}^i \zeta^A f_{iA}^X, \quad (3.64)$$

$$\delta_\varepsilon \zeta^A = -\frac{i}{2} \gamma^a \varepsilon^i \widehat{\mathcal{D}}_a q^X f_{iX}^A - \delta_\varepsilon q^X \omega_{X B}{}^A \zeta^B + g \varepsilon^i \mathcal{N}_i^A. \quad (3.65)$$

The hatted quantities $\widehat{}$ are the supercovariantization of the unhatted ones.

4 The action

We now turn to the Lagrangean of the $\mathcal{N} = 2$, $D = 5$ gauged supergravity in interaction with vector, tensor and hypermultiplets. We take as a start the results of [5, 10], and add all the modifications that are needed in presence of hypermultiplets and $SU(2)_R$ gauging. We'll exhibit our result at the *component level*, and for brevity, whenever we use the same symbols as in section 3, we now mean those objects evaluated at $\vartheta = 0 = d\vartheta$. In particular, the superspace differential d now becomes the ordinary differential.

Each form can be decomposed along the vielbeins $e^a = dx^\mu e_\mu{}^a$ and the gravitino, which reduces to $\psi^i = dx^\mu \psi_\mu^i \equiv e^a \psi_a^i$. The supercovariant connection one-form $\hat{\omega}_a{}^b = dx^\mu \hat{\omega}_{\mu a}{}^b$ is naturally introduced, via equation (3.41), now evaluated at $\vartheta = 0 = d\vartheta$, as

$$de^a + e^b \hat{\omega}_b{}^a = -\frac{1}{4} \bar{\psi}^i \gamma^a \psi_i. \quad (4.1)$$

This determines ω as the metric connection, augmented by the standard gravitino bilinears. The supercovariant curvature two-form becomes $R_a{}^b = d\hat{\omega}_a{}^b + \hat{\omega}_a{}^c \hat{\omega}_c{}^b$, with $\hat{\omega}$ given in (4.1).

Since we write the Lagrangean as a five-form, it is also convenient to define the $(5-p)$ -forms

$$\hat{e}^{a_1 \dots a_p} \equiv -\frac{1}{(5-p)!} \epsilon^{a_1 \dots a_p b_1 \dots b_{5-p}} e_{b_1} \dots e_{b_{5-p}}. \quad (4.2)$$

In particular $\hat{e} = \sqrt{-g} d^5 x$.

Invariance of the action under supersymmetry can be checked by using the standard trick of lifting this action to superspace, performing the superspace differential and taking the interior product with ε^A . It is obvious that the equation (2.8) gives

$$\delta_\varepsilon \int \mathcal{L} = \int i_\varepsilon \mathcal{D} \mathcal{L} + \int \mathcal{D} i_\varepsilon \mathcal{L}$$

and $\mathcal{D} i_\varepsilon \mathcal{L}$ is a total derivative that we can discard.

The Lagrangean for the gauged theory can be split as

$$\mathcal{L} = \mathcal{L}_{KIN} + \mathcal{L}_{Pauli} + \mathcal{L}_{CS} + \mathcal{L}_{mass} + \mathcal{L}_{pot} + \mathcal{L}_{4Fermi},$$

where \mathcal{L}_{KIN} contains the kinetic terms, \mathcal{L}_{Pauli} describes the couplings between the bosonic field-strengths and the fermions, \mathcal{L}_{CS} contains the Yang-Mills Chern-Simons term, \mathcal{L}_{mass} gives the mass of the fermions, \mathcal{L}_{pot} contains the typical potential of the gauged theories and \mathcal{L}_{4Fermi} contains the four-Fermi terms.

The kinetic terms are

$$\begin{aligned} \mathcal{L}_{KIN} = & \frac{1}{2} R^{ab} \hat{e}_{ab} + \frac{i}{4} e^a e^b \bar{\psi}^i \gamma_{ab} \mathcal{D} \psi_i - \frac{1}{2} \bar{\lambda}^{i\tilde{a}} \gamma_a \mathcal{D} \lambda_i^{\tilde{a}} \hat{e}^a - \bar{\zeta}^A \gamma^a \mathcal{D} \zeta_A \hat{e}_a + \\ & + \left[\frac{1}{2} g_{\tilde{x}\tilde{y}} Q_a^{\tilde{x}} Q^{a\tilde{y}} \hat{e} - \hat{e}^a g_{\tilde{x}\tilde{y}} Q_a^{\tilde{x}} \left(\mathcal{D} \phi^{\tilde{y}} - \frac{i}{2} \bar{\psi}^i \lambda_i^{\tilde{a}} f_{\tilde{a}}^{\tilde{y}} \right) \right] + \\ & + \left[\frac{1}{2} g_{XY} Q_a^X Q^{aY} \hat{e} - \hat{e}^a g_{XY} Q_a^X \left(\mathcal{D} q^Y + i \bar{\psi}^i \zeta^A f_{iA}^Y \right) \right] + \\ & + \frac{1}{4} a_{\tilde{I}\tilde{J}} Q_{ab}^{\tilde{I}} \left[Q^{\tilde{J}ab} \hat{e} + 2 \hat{e}^{ab} \left(\mathcal{H}^{\tilde{J}} + \frac{1}{2} e^c \bar{\psi}^i \gamma_c \lambda_i^{\tilde{a}} h_{\tilde{a}}^{\tilde{J}} - \frac{i\sqrt{6}}{8} \bar{\psi}^i \psi_i h^{\tilde{J}} \right) \right] + \\ & + \frac{1}{g} \Omega_{MN} B^M H^N, \end{aligned} \quad (4.3)$$

where $Q_a^{\tilde{x}}$, Q_a^A and $Q_{ab}^{\tilde{I}}$ are auxiliary fields which have been introduced in order to write the Lagrangean as a five-form. Their equations of motion give

$$\begin{aligned} e^a Q_a^{\tilde{x}} &= \mathcal{D} \phi^{\tilde{x}} - \frac{i}{2} \bar{\psi}^i \lambda_i^{\tilde{a}} f_{\tilde{a}}^{\tilde{x}}, \\ e^a Q_a^X &= \mathcal{D} q^X + i \bar{\psi}^i \zeta^A f_{iA}^X, \\ \frac{1}{2} e^a e^b Q_{ba}^{\tilde{I}} &= \mathcal{H}^{\tilde{I}} + \frac{1}{2} e^c \bar{\psi}^i \gamma_c \lambda_i^{\tilde{a}} h_{\tilde{a}}^{\tilde{I}} - \frac{i\sqrt{6}}{8} \bar{\psi}^i \psi_i h^{\tilde{I}}, \end{aligned} \quad (4.4)$$

which, upon substituting this back in \mathcal{L}_{Kin} , yields the usual super-covariantized kinetic terms.

The Pauli-like couplings are described by:

$$\begin{aligned}\mathcal{L}_{Pauli} &= \frac{i}{4} \bar{\lambda}^{i\tilde{a}} \gamma_{ab} \psi_i \mathcal{H}^{\tilde{I}} h_{\tilde{I}}^{\tilde{a}} e^a e^b - \frac{\sqrt{6}}{8} \bar{\psi}^i \gamma_a \psi_i \mathcal{H}^{\tilde{I}} h_{\tilde{I}} e^a - \frac{i}{4} \bar{\lambda}_{\tilde{a}}^i \gamma_{ab} \psi_i \mathcal{D} \phi^{\tilde{x}} f_{\tilde{x}}^{\tilde{a}} \hat{e}^{ab} + \\ &- \frac{i}{4} \Phi_{\tilde{I}\tilde{a}\tilde{b}} \bar{\lambda}^{i\tilde{a}} \gamma_{ab} \lambda_i^{\tilde{b}} \mathcal{H}^{\tilde{I}} \hat{e}^{ab} + \frac{i}{4\sqrt{6}} \zeta_A \gamma_{ab} \zeta^A \mathcal{H}^{\tilde{I}} h_{\tilde{I}} \hat{e}^{ab} + \\ &+ \frac{i}{2} \bar{\psi}^i \gamma_{ab} \zeta^A f_{Ai}^X \mathcal{D} q^Y g_{XY} \hat{e}^{ab},\end{aligned}\tag{4.5}$$

where

$$\Phi_{\tilde{I}\tilde{a}\tilde{b}} \equiv \sqrt{\frac{2}{3}} \left(\frac{1}{4} \delta_{\tilde{a}\tilde{b}} h_{\tilde{I}} + T_{\tilde{a}\tilde{b}\tilde{c}} h_{\tilde{I}}^{\tilde{c}} \right),$$

whereas the Chern-Simons couplings are fixed as usual to

$$\mathcal{L}_{CS} = \frac{2}{3\sqrt{6}} C_{IJK} \left(F^I F^J A^K - \frac{3}{4} g F^I A^J A^L A^F f_{LF}^K + \frac{3}{20} g^2 f_{GH}^J f_{LF}^K A^I A^G A^H A^L A^F \right). \tag{4.6}$$

The four Fermi terms can further be split, following

$$\mathcal{L}_{4Fermi} = \mathcal{L}_{4\lambda} + \mathcal{L}_{3\lambda\psi} + \mathcal{L}_{other},$$

such that

$$\begin{aligned}\mathcal{L}_{4\lambda} &= \hat{e} \left[\frac{1}{48\sqrt{6}} \bar{\lambda}^{i\tilde{a}} \gamma_{ab} \lambda_i^{\tilde{b}} \bar{\lambda}^{j\tilde{c}} \gamma^{ab} \lambda_j^{\tilde{d}} T_{\tilde{a}\tilde{b}\tilde{c};\tilde{d}} + \frac{1}{24} K_{\tilde{a}\tilde{b}\tilde{c}\tilde{d}} \left(2 \bar{\lambda}^{i\tilde{a}} \lambda_i^{\tilde{b}} \bar{\lambda}^{j\tilde{c}} \lambda_j^{\tilde{d}} + \right. \right. \\ &+ \left. \bar{\lambda}^{i\tilde{a}} \gamma_a \lambda_i^{\tilde{b}} \bar{\lambda}^{j\tilde{c}} \gamma^a \lambda_j^{\tilde{d}} \right) - \frac{1}{12} \bar{\lambda}^{i\tilde{a}} \lambda^{j\tilde{a}} \bar{\lambda}_j^{\tilde{b}} \lambda_i^{\tilde{b}} + \\ &- \left. \frac{1}{24} \bar{\lambda}^{i\tilde{a}} \gamma_a \lambda^{j\tilde{a}} \bar{\lambda}_j^{\tilde{b}} \gamma^a \lambda_i^{\tilde{b}} + \frac{1}{64} \bar{\lambda}^{i\tilde{a}} \gamma_{ab} \lambda^{j\tilde{a}} \bar{\lambda}_j^{\tilde{b}} \gamma^{ab} \lambda_i^{\tilde{b}} \right]\end{aligned}\tag{4.7}$$

and

$$\mathcal{L}_{3\lambda\psi} = \frac{2i}{3\sqrt{6}} T_{\tilde{a}\tilde{b}\tilde{c}} \left[\bar{\lambda}^{i\tilde{a}} \psi^j \bar{\lambda}_i^{\tilde{b}} \gamma_a \lambda_j^{\tilde{c}} - \frac{1}{2} \bar{\lambda}^{i\tilde{a}} \gamma_a \psi^j \bar{\lambda}_i^{\tilde{b}} \lambda_j^{\tilde{c}} \right] \hat{e}^a \tag{4.8}$$

exactly reproduce the terms of the Einstein-Maxwell theory presented in [5].

\mathcal{L}_{other} contains the new four-Fermi interactions with the hypermultiplet spinors and reads:

$$\begin{aligned}\mathcal{L}_{other} &= -\frac{i}{32} e^a \bar{\psi}^i \gamma_a \psi_i \bar{\psi}^j \psi_j + \hat{e}_{ab} \left[\frac{1}{32} \bar{\psi}^i \psi_i \bar{\zeta}_A \gamma^{ab} \zeta^A + \frac{1}{4} \bar{\zeta}_A \gamma^{ab} \psi_i \bar{\psi}^i \zeta^A \right] + \\ &+ \left[\frac{1}{16} \bar{\lambda}^{\tilde{a}i} \gamma_{ab} \psi_i \bar{\lambda}^{\tilde{a}k} \psi_k + \frac{1}{16} \bar{\lambda}^{\tilde{a}i} \gamma^c \psi_i \bar{\lambda}^{\tilde{a}j} \gamma_{abc} \psi_j - \frac{1}{64} \bar{\lambda}^{\tilde{a}i} \gamma_{ab} \lambda_{\tilde{a}i} \bar{\psi}^k \psi_k \right] \hat{e}^{ab} + \\ &+ \hat{e} \frac{1}{16} \bar{\zeta}_A \gamma^{ab} \zeta^A \bar{\zeta}_B \gamma_{ab} \zeta^B - \frac{1}{4} \Omega_{ABCD} \left(5 \bar{\zeta}^A \zeta^B \bar{\zeta}^C \zeta^D - \bar{\zeta}^A \gamma^a \zeta^B \bar{\zeta}^C \gamma_a \zeta^D \right) \\ &+ \hat{e} \frac{1}{16} \bar{\zeta}_A \gamma^{abc} \zeta^A \bar{\lambda}^{\tilde{a}i} \gamma_{abc} \lambda_{\tilde{a}i}.\end{aligned}\tag{4.9}$$

We point out that there are no terms with three gravitinos and one gluino or two ζ , one gluino and one gravitino due to the orthogonality properties of $h^{\tilde{I}}$ and $h^{\tilde{I}\tilde{a}}$.

We can finally describe the mass and potential terms. The first reads

$$\begin{aligned}\mathcal{L}_{mass} = & g_R \left[\frac{i}{2\sqrt{6}} \bar{\lambda}^{i\tilde{a}} \lambda^{j\tilde{b}} P_{ij}^{\tilde{a}\tilde{b}} \hat{e} - \bar{\lambda}^{i\tilde{a}} \gamma_a \psi^j P_{ij}^{\tilde{a}} \hat{e}^a - \frac{i\sqrt{6}}{8} \bar{\psi}^i \gamma_{ab} \psi^j P_{ij} \hat{e}^{ab} \right] + \\ & + g \left[\bar{\lambda}^{i\tilde{a}} \gamma_a \psi_i W^{\tilde{a}} \hat{e}^a + \bar{\lambda}^{i\tilde{a}} \lambda_j^{\tilde{b}} W_{\tilde{a}\tilde{b}} \hat{e} + 2 \bar{\psi}^i \gamma_a \zeta^A \mathcal{N}_{iA} \hat{e}^a + \right. \\ & \left. - 2i \bar{\zeta}^A \lambda^{\tilde{a}i} \mathcal{M}_{Ai\tilde{a}} \hat{e} + \bar{\zeta}^A \zeta^B \mathcal{M}_{AB} \hat{e} \right],\end{aligned}\quad (4.10)$$

where $P_{ij}^{\tilde{a}}$, P_{ij} , $W^{\tilde{a}}$ and \mathcal{N}_{iA} were defined in the previous section by equations (3.47), (3.43), (3.48) and (3.56), and the other mass matrices are defined as

$$P_{ij}^{\tilde{a}\tilde{b}} \equiv \delta^{\tilde{a}\tilde{b}} P_{ij} + 4T^{\tilde{a}\tilde{b}\tilde{c}} P_{ij}^{\tilde{c}}, \quad (4.11)$$

$$W^{\tilde{a}\tilde{b}} \equiv i h^{I[\tilde{a}} K_I^{\tilde{b}]} + \frac{i\sqrt{6}}{4} h^I K_I^{\tilde{a};\tilde{b}}, \quad (4.12)$$

$$\mathcal{M}_{Ai\tilde{a}} \equiv f_{AiX} K_I^X h^{I\tilde{a}}, \quad (4.13)$$

$$\mathcal{M}_{AB} \equiv \frac{i\sqrt{6}}{2} f_{AX}^i f_{BiY} K_I^{[Y;X]} h^I. \quad (4.14)$$

This mass term is of first order in the gauge coupling constants and has coefficients fixed by variations of the kinetic terms.

Finally, the potential is ($\mathcal{L}_{Pot} = -\mathcal{V}\hat{e}$):

$$\mathcal{V} = 2g^2 W^{\tilde{a}} W_{\tilde{a}} - g_R^2 \left[2P_{ij} P^{ij} - P_{ij}^{\tilde{a}} P^{\tilde{a}ij} \right] + 2g^2 \mathcal{N}_{iA} \mathcal{N}^{iA}. \quad (4.15)$$

As an outcome of this complete analysis, one can remark many similarities with the analogous four-dimensional matter coupled theory [34]. However, a first difference is the existence of tensor multiplets satisfying a first order equation of motion, and of a corresponding new term in the scalar potential. The second lies in the presence of the Chern–Simons term, that as well known is a peculiar feature of odd-dimensional space–times. Moreover, the geometry described by the scalars of vector multiplets is now “very special” rather than special Kähler, and thus the $U(1)$ Kähler connection does not exist and all the 4D structures deriving from its gauging are missing. Finally, as already remarked, the Yang–Mills gauging in $D = 5$ does not give rise to any contribution to the scalar potential unless some of the ungauged vectors are dualised into tensor multiplets.

5 Some comments on the scalar potential

We have found that the bosonic sector of 5D, $\mathcal{N} = 2$ supergravity is described by the Lagrangean:

$$\begin{aligned}\hat{e}^{-1} \mathcal{L}_{bosonic}^{\mathcal{N}=2} = & -\frac{1}{2} R - \frac{1}{4} a_{\tilde{I}\tilde{J}} \mathcal{H}_{\mu\nu}^{\tilde{I}} \mathcal{H}^{\tilde{J}\mu\nu} - \frac{1}{2} g_{XY} \mathcal{D}_\mu q^X \mathcal{D}^\mu q^Y + \\ & - \frac{1}{2} g_{\tilde{x}\tilde{y}} \mathcal{D}_\mu \phi^{\tilde{x}} \mathcal{D}^\mu \phi^{\tilde{y}} + \frac{\hat{e}^{-1}}{6\sqrt{6}} C_{IJK} \epsilon^{\mu\nu\rho\sigma\tau} F_{\mu\nu}^I F_{\rho\sigma}^J A_\tau^K + \\ & + \frac{\hat{e}^{-1}}{4g} \epsilon^{\mu\nu\rho\sigma\tau} \Omega_{MN} B_{\mu\nu}^M \mathcal{D}_\rho B_{\sigma\tau}^N - \mathcal{V}(\phi, q),\end{aligned}\quad (5.1)$$

where

$$\mathcal{V} = 2g^2 W^{\tilde{a}} W^{\tilde{a}} - g_R^2 [2P_{ij} P^{ij} - P_{ij}^{\tilde{a}} P^{\tilde{a}ij}] + 2g^2 \mathcal{N}_{iA} \mathcal{N}^{iA}. \quad (5.2)$$

The bosonic part of the supersymmetry transformation rules is given by

$$\delta_\varepsilon \psi_{\mu i} = \mathcal{D}_\mu(\hat{\omega}) \varepsilon_i + \frac{i}{4\sqrt{6}} h_{\tilde{I}} (\gamma_{\mu\nu\rho} \varepsilon_i - 4g_{\mu\nu} \gamma_\rho \varepsilon_i) \widehat{\mathcal{H}}^{\nu\rho\tilde{I}} + \frac{i}{\sqrt{6}} g_R \gamma_\mu \varepsilon^j P_{ij}, \quad (5.3)$$

$$\delta_\varepsilon \lambda_i^{\tilde{a}} = -\frac{i}{2} f_{\tilde{x}}^{\tilde{a}} \gamma^\mu \varepsilon_i \widehat{\mathcal{D}}_\mu \phi^{\tilde{x}} + \frac{1}{4} h_{\tilde{I}}^{\tilde{a}} \gamma^{\mu\nu} \varepsilon_i \widehat{\mathcal{H}}_{\mu\nu}^{\tilde{I}} + g_R \varepsilon^j P_{ij}^{\tilde{a}} + g W^{\tilde{a}} \varepsilon_i, \quad (5.4)$$

$$\delta_\varepsilon \zeta^A = -\frac{i}{2} f_{iX}^A \gamma^\mu \varepsilon^i \widehat{\mathcal{D}}_\mu q^X + g \varepsilon^i \mathcal{N}_i^A, \quad (5.5)$$

where

$$P_{ij} \equiv h^I P_{Iij}, \quad (5.6)$$

$$P_{ij}^{\tilde{a}} \equiv h^{\tilde{a}I} P_{Iij}, \quad (5.7)$$

$$W^{\tilde{a}} \equiv -\frac{\sqrt{6}}{8} \Omega^{MN} h_{\tilde{M}}^{\tilde{a}} h_N = \frac{\sqrt{6}}{4} h^I K_I^{\tilde{x}} f_{\tilde{x}}^{\tilde{a}}, \quad (5.8)$$

$$\mathcal{N}^{iA} \equiv \frac{\sqrt{6}}{4} h^I K_I^X f_X^{Ai}. \quad (5.9)$$

As expected, one sees that the scalar potential of the gauged supergravity theory is constructed out of the squares of the fermion shifts that arise in the supersymmetry transformations due to the gauging.

The four terms in the potential (5.2) have different origins. Those of order g_R^2 come from the R -symmetry gauging, whereas those of order g^2 come from the gauging of the Yang–Mills group K . In detail, the YM ones are given by the squares of the g order shifts in the supersymmetry transformation laws of the $\lambda_i^{\tilde{a}}$ and ζ^A fields, while the R -symmetry ones are given by the square of the order g_R shifts in the ψ^i and $\lambda_i^{\tilde{a}}$ supersymmetry variations. As in the four-dimensional case [34], this can be related to the existence of a “Ward identity” for the scalar potential [35], but here we don’t need to use the (3.15) condition on the prepotential to ensure it, due to the reality properties of the very special manifold \mathcal{S} parametrised by the h^I coordinates.

In presence of hypermultiplets, one generically has in the fermionic shifts the $SU(2)_R$ -valued quantities $P_{ij}(\phi, q)$ and $P_{ij}^{\tilde{a}}(\phi, q)$ of (5.6) and (5.7) containing the prepotential

$$P_{Iij} \equiv iP_I^r(q)(\sigma_r)_{ij} \quad r = 1, 2, 3, \quad (5.10)$$

where $(\sigma_r)_i^j$ are the usual Pauli matrices, in place of the $P_0 \delta_{ij}$ and $P^{\tilde{a}} \delta_{ij}$ of [6, 10, 11].

For our metric signature, this potential allows for the existence of Anti de Sitter vacua if $\mathcal{V}(q^*, \phi^*) < 0$ for $\mathcal{V}'(q^*, \phi^*) = 0$. Thus it is straightforward to see that the only contribution which can allow for such solutions is the $2P_{ij} P^{ij}$ term, coming from the R -symmetry gauging of the gravitinos. This implies that a simple Yang–Mills gauging, even in presence of both tensor and hypermultiplets, does not allow Anti de Sitter solutions.

Since (5.2) is the most general potential of $5D$, $\mathcal{N} = 2$ gauged supergravity coupled to all matter, all the previously studied examples must be found as peculiar subcases.

We first analyze the choice $n_H = 0$. Since in this case the \mathcal{Q} -manifold disappears, we are forced to put $K_I^X = 0$, and the Yang–Mills sector is reduced to the W^2 part due to the tensor multiplets. Moreover, the absence of the quaternionic fields q^X implies that the prepotentials are set to zero, or at most are $SU(2)$ -valued constants. Their most general form is now given by

$$P_{Ii}{}^j = i\xi_I^r (\sigma_r)_{i}{}^j, \quad (5.11)$$

where ξ_I^r are three real constants generically⁷ breaking $SU(2) \rightarrow U(1)$.

The (3.15) condition becomes

$$g f_{IJ}^K \xi_K^r = g_R \epsilon^{rst} \xi_I^s \xi_J^t. \quad (5.12)$$

If one makes the choice $\xi_I = (0, V_I, 0)$, the condition (5.12) reduces to

$$f_{IJ}^K V_K = 0, \quad (5.13)$$

which is the supersymmetry requirement of [10]. In particular, all the results therein can be recovered by substituting

$$P_{Iij} = V_I \delta_{ij} \quad (5.14)$$

in the action and supersymmetry laws. The above mechanism is the local analogue of the Fayet–Iliopoulos phenomenon occurring also in four dimensions [34].

The $n_T = 0$ case is trivial, as it simply removes the $g^2 W^2$ term. This leaves us with the potential presented in [9], where the gauge group was chosen as $K = U(1)^{n_V+1}$ and $g_R = g$.

More interesting is to take $n_V = 0$. This implies that there is only one vector: the graviphoton. The potential is still non-vanishing, and becomes

$$\mathcal{V} = \frac{3}{4} g^2 K^X K_X - 2g_R^2 P_{ij} P^{ij}, \quad (5.15)$$

which in principle could admit Anti de Sitter vacua.

The $n_V = n_T = n_H = 0$ case gives back the pure gauged supergravity of [4], with the potential

$$\mathcal{V} = -4V^2, \quad (5.16)$$

for the choice $P_{ij} = V \delta_{ij}$ ($\xi = (0, V, 0)$). This is, of course, the Anti de Sitter five-dimensional supergravity.

It is now relevant to consider the possible existence of smooth Randall–Sundrum domain–wall solutions of type RS2 providing an “alternative to compactification”. An

⁷ In some special cases [43], one can still preserve the full $SU_R(2)$ gauging by the choice $\xi_I^r = \delta_I^r$ and thus identifying by (5.12) the $SU_R(2)$ structure constants with those of a $SU(2) \subset K$: $g f_{st}^r = g_R \epsilon^{rst}$.

easy way to do it is to determine the vacua obtained from the full potential (5.2) and check whether they are of the same nature as those already studied in [6, 27, 11].

It has been shown [27] that in absence of tensor multiplets there are no RS2 solutions at all, while if tensor multiplets are added one still excludes the presence of supersymmetric solutions but leaves open the possibility of having non-BPS ones.

Although we have not yet performed a complete analysis of (5.2), we can already make some comments on the supersymmetric vacua and BPS solutions of the full theory.

As for the theory without hypermultiplets [27, 11], the cosmological constant of an $\mathcal{N} = 2$ supersymmetric vacuum is only given by

$$\mathcal{V}(\phi^*, q^*) = g_R^2 P_{ij} P^{ij}(\phi^*, q^*). \quad (5.17)$$

Using the orthogonality property

$$W^{\tilde{a}} P_{ij}^{\tilde{a}} = 0,$$

one can easily show that the requirement $\langle \delta_\varepsilon \lambda \rangle = \langle \delta_\varepsilon \zeta \rangle = 0$ implies that an $\mathcal{N} = 2$ supersymmetric ground state must satisfy

$$\langle W^{\tilde{a}} \rangle = \langle P_{ij}^{\tilde{a}} \rangle = \langle \mathcal{N}_{iA} \rangle = 0. \quad (5.18)$$

Therefore, the only non-trivial effect of the $W^{\tilde{a}}$ and \mathcal{N}_{iA} terms can be a change in the shape of the critical point.

The above result can also be obtained by a different argument⁸. The integrability condition on the gravitino supersymmetry rule (5.3) imposes that the vacuum expectation value is given by (5.17). Thus, in order to have non vanishing $\langle W^{\tilde{a}} \rangle$, $\langle P_{ij}^{\tilde{a}} \rangle$ and $\langle \mathcal{N}_{iA} \rangle$ they must compensate each other in the potential. However, being all positive squares, they can never cancel unless they vanish. This means that the $\mathcal{N} = 2$ supersymmetric critical points of the full $\mathcal{N} = 2$, $D = 5$ gauged supergravity theory have the same nature as those of the reduced theory analyzed in [27].

There remains to examine the possibility of having lower supersymmetric BPS solutions.

To do this, one tries to relax the conditions (5.18) near the critical point, where scalars are not fixed anymore. It has been verified that in absence of tensor and hypermultiplets the derivative of scalars in the radial direction y can be chosen proportional to the derivative of the superpotential, leading to solutions that preserve half supersymmetry. When tensor multiplets are added, it turns out [27] that relaxing the condition $W^{\tilde{a}} = 0$, forbids any supersymmetric solution.

We now show along the lines of [27] that the same phenomenon occurs also in the most general case where hypermultiplets are added and the full $SU(2)_R$ group is gauged.

⁸We are indebted to R. Kallosh for explaining this argument and for important discussions on the results below.

We choose the y dependence of all fields appearing in the gluino susy rule (5.4) as

$$\partial_y \phi^{\tilde{a}}(y) \sim W^{\tilde{a}}(\phi) \sim P_{ij}^{\tilde{a}}(\phi, q).$$

For BPS solutions to exist, one has to find some Killing spinors which, in addition to the usual constraint coming from the gravitino susy rule (5.3)

$$i\gamma^y \varepsilon_i \sim g_R P_{ij} \varepsilon^j, \quad (5.19)$$

also satisfy

$$\delta_\varepsilon \lambda_i^{\tilde{a}} = \mathcal{A}^{\tilde{a}}_i{}^j \varepsilon_j = 0 \quad (5.20)$$

where the operator matrix $\mathcal{A}^{\tilde{a}}$ can be read off from (5.4). This amounts to finding a Killing spinor eigenvector of $\mathcal{A}^{\tilde{a}}$ with zero eigenvalue, i.e. the matrix $\mathcal{A}^{\tilde{a}}$ must be degenerate. It can be seen that even if with the full R-symmetry gauging the P_{ij} and $P_{ij}^{\tilde{a}}$ are $SU_R(2)$ -valued matrices, this condition still has no solutions. Indeed, upon substituting (5.19) into the gluino transformation rule (5.4), and requiring that all bosonic functions of the scalars have the same y behaviour, the $\mathcal{A}^{\tilde{a}}$ operator reduces to the form

$$\mathcal{A}^{\tilde{a}}_i{}^j \sim iQ^{r\tilde{a}}(\sigma_r)_i{}^j + W^{\tilde{a}}\delta_i^j, \quad (5.21)$$

where $Q^{r\tilde{a}}$ indicates some real combination of the various y -dependent quantities

$$Q^{r\tilde{a}}(y) \sim \partial_y \phi^{\tilde{a}} P^r + P^{r\tilde{a}}, \quad (5.22)$$

and the obvious notation $P_{ij} \equiv iP^r(\sigma_r)_{ij}$, $P_{ij}^{\tilde{a}} \equiv iP^{r\tilde{a}}(\sigma_r)_{ij}$. Requiring $\det \mathcal{A}^{\tilde{a}} = 0$ (or equivalently for a projector $\mathcal{A}^2 = \mathcal{A}$), imposes for each \tilde{a}

$$(W^{\tilde{a}})^2 + (Q^{\tilde{a}})^2 = 0, \quad (5.23)$$

which has no solutions except for $W^{\tilde{a}} = 0 = Q^{\tilde{a}}$ and this takes us back to the cases of [27].

This seems to rule out the presence of BPS solutions, at least when $h_{\tilde{I}}\mathcal{H}^{\tilde{I}} = h_{\tilde{I}}^{\tilde{a}}\mathcal{H}^{\tilde{I}} = 0$. The only open possibilities appear to be either BPS solutions with non-trivial electric or magnetic fields $\mathcal{H}^{\tilde{I}}$, or non-BPS solutions. It must be noted, however, that also the examples considered up to now with $\mathcal{H}^{\tilde{I}} \neq 0$ do not seem to admit solutions of the RS2 type [41], i.e. with a metric of the form

$$ds^2 = a(y) dx^2 + dy^2,$$

approaching asymptotically AdS_5 .

We leave the investigation of all these more general cases to future work.

6 The low-energy theory for the $AdS_5 \times T^{11}$ compactification

We collect here, as a further application of the matter coupled 5D gauged supergravity, some comments about the structure of the theory corresponding to the type IIB compactification on $AdS_5 \times T^{11}$ [17]. This study could reveal very useful for further analyzing the AdS/CFT correspondence in such non-trivial case.

One is interested in the construction of the low-energy theory in order to study the deformations and RG fluxes of the dual field theory. Once a specific scalar manifold \mathcal{M} is chosen, one can study the stationary points of the potential and look for solutions interpolating between them. This should correspond to a flux in the CFT side.

As pointed out by S. Ferrara, a very interesting deformation of the four-dimensional conformal field theory [16] is given by the operator

$$Tr(A_i B_j A_k B_l) (\sigma^r)^{ik} (\sigma^s)^{jl} \delta_{rs}, \quad (6.1)$$

which has conformal dimension $\Delta = 3$ and therefore corresponds to a marginal deformation preserving supersymmetry.

The generic superpotential now is given by

$$\mathcal{W} = \left[\lambda \epsilon^{ij} \epsilon^{kl} + \mu (\sigma^r)^{ik} (\sigma_r)^{jl} \right] Tr(A_i B_j A_k B_l), \quad (6.2)$$

where λ and μ are two coupling constants and the isometry group has now been broken from $SU(2) \times SU(2)$ to the diagonal $SU(2)$.

From the supergravity point of view, this implies that there should exist another $\mathcal{N} = 2$ vacuum of five-dimensional supergravity with this symmetry group. Lifting this solution to ten dimensions should give a metric which is the warped product of AdS_5 and a deformation of T^{11} with isometry reduced to $SU(2)_{diag}$.

An important point is that the T^{11} compactification does not seem to be a solution of the $\mathcal{N} = 8$ theory [42]. If such solution would exist, we should find a stable vacuum with $G = SU(2) \times SU(2)$ isometry preserving $\mathcal{N} = 2$ supersymmetry. There are two possible embeddings of G in the $\mathcal{N} = 8$ gauge group $SO(6) \simeq SU(4)$, leading to two families of vacua:

$$i) \quad 6 \rightarrow (2, 2) + (1, 1) + (1, 1),$$

$$ii) \quad 6 \rightarrow (3, 1) + (1, 3).$$

The first family can be obtained by turning on simultaneously the two scalars λ and μ , which break $SO(6) \rightarrow SO(4) \times SO(2)$ and $SO(6) \rightarrow SO(5)$. According to the analysis of [42], this gives a supersymmetry operator of the form

$$W_{ab} = -\frac{1}{4} \left(4e^{\lambda+\mu} + e^{\mu-\lambda} + e^{-\lambda-\mu} \right) \delta_{ab}, \quad (6.3)$$

in the gravitinos supersymmetry transformations, which cannot preserve $\mathcal{N} = 2$. In fact, in order to preserve \mathcal{N} supersymmetries, the operator W_{ab} must have, at the critical point, \mathcal{N} eigenvalues of the form

$$\pm \sqrt{-\frac{3}{g^2} \mathcal{V}|_*}. \quad (6.4)$$

To obtain vacua of the type (ii), we must study the potential given by the three scalars respecting this invariance [42]. We find that the potential becomes in this case

$$\mathcal{V} = -\frac{3}{8}g^2 \left[3f^2(\lambda)(\alpha^2 + \beta^2) \sinh^2 \lambda + 3 \cosh^2(2\lambda) - \cosh(4\lambda) \right], \quad (6.5)$$

where λ, α, β are the three scalars and $f(\lambda) \equiv \frac{e^\lambda - e^{-2\lambda}}{3\lambda}$. At the extremum one finds the potential $\mathcal{V} = -\frac{3}{4}g^2$ and the supersymmetry operator $W_{ab} = -\frac{3}{2}\delta_{ab}$. Using (6.4) one finds that eight supersymmetries are preserved, and one retrieves the highest symmetric S^5 solution.

In conclusion, the aforementioned flux from the T^{11} to a solution with residual $SU(2)_{diag}$ symmetry can only be studied within the $\mathcal{N} = 2, D = 5$ gauged theory corresponding to the T^{11} low-energy model.

To build this model, we need all the predictive power of the *AdS/CFT* correspondence and the spectrum analysis performed in [17]. As for the maximally symmetric cases of type IIB on $AdS_5 \times S^5$ or *M*-theory on $AdS_{4/7} \times S^{7/4}$, we expect that the low-energy gauged supergravity states correspond to the theory given by fields which are the products of two singletons. This means that their masses are less than or at most equal to zero.

Using the [17] results and notations, the theory should be described by the massless graviton multiplet, corresponding to the stress-energy tensor $W_\alpha \bar{W}_{\dot{\alpha}} + \dots$ from the boundary point of view, the seven massless vector multiplets corresponding to the $SU_A(2) \times SU_B(2) \times U_{Betti}(1)$ conserved currents $A_i \bar{A}_j, B_i \bar{B}_j$ and $Tr(A_i \bar{A}^i + B_i \bar{B}^i)$, and six hypermultiplets corresponding to the $Tr(AB)$ and W^2 operators.

There are no tensor multiplets, since they should be described by the product of at least three singleton states $Tr[W_\alpha(AB)]$.

Although this is not enough to uniquely fix the scalar manifold $\mathcal{M} = \mathcal{S} \otimes \mathcal{Q}$, we can still extract some useful information. For example, the \mathcal{Q} manifold must contain at least the $G = SU(2) \times SU(2)$ isometries and two zero-modes: i.e. two of its scalars must be massless, since they have to correspond to the moduli of the conformal field theory. The correct manifold must anyhow fall in the classification of [32].

We can say something more on the \mathcal{S} manifold of the vector multiplet scalars. As already claimed in [17], the *AdS/CFT* foresees the value of the Chern-Simons couplings $C_{\bar{I}\bar{J}\bar{K}}$ through the computation of the anomalies in the boundary theory. The result is that the polynomial (3.3) describing the \mathcal{S} target manifold is given by

$$\alpha \xi_r^3 + \beta \xi_r(\xi_A^2 + \xi_B^2) + \gamma \xi_r \xi_b^2 + \delta \xi_b(\xi_A^2 - \xi_B^2) = 1, \quad (6.6)$$

where α, β, γ and δ are constants and ξ_r, ξ_b, ξ_A and ξ_B denote the ambient coordinates corresponding to the R -symmetry, Betti and $SU_A(2) \times SU_B(2)$ symmetries.

It is known that there always exists a point c^I where the metric becomes flat (δ_{IJ}) and the cubic polynomial takes the standard form. In our case this happens for $c_r = 1$ and $c_A = c_B = c_b = 0$ and fixes

$$\alpha = 1, \quad \beta = \gamma = -\frac{1}{2}, \quad (6.7)$$

whereas δ is still free.

At this point one can go farther and see whether this manifold corresponds to one of the homogeneous spaces classified in [32]. Unfortunately, it is easy to prove that the given polynomial cannot correspond to a homogeneous space, as there is no $SO(7)$ rotation reducing the δ piece of (6.6) to the form of one of the three families classified in [32].

Anyway, this does not spoil our hope to study the minima of the potential for such a model in the future, since the cubic surface is specified by (6.6) and (6.7) up to the δ coefficient, which can be computed explicitly evaluating the three-point functions of the corresponding anomalies.

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Appendix A: Notations and conventions

The five-dimensional superspace is spanned by the supercoordinates $Z^\mu = (x^\mu, \theta^{mi})$, where x^μ ($\mu = 0, \dots, 4$) are the ordinary space-time coordinates and θ^{mi} ($m = 1, \dots, 4$) are symplectic-Majorana spinors carrying the $USp(2)$ doublet index $i = 1, 2$ which is raised and lowered with the invariant $USp(2)$ tensor $\epsilon_{12} = \epsilon^{12} = 1$ as follows:

$$\vartheta^{mi} = \epsilon^{ij} \vartheta_{mj}, \quad \vartheta_i^m = \vartheta^{mj} \epsilon_{ji}. \quad (A.1)$$

The flat superspace indices are $\underline{a} = (a\alpha)$. The vector ones are raised and lowered with the flat metric $\eta_{ab} = \{-++++\}$.

The symplectic-Majorana condition on a generic spinor $\lambda_{\alpha i}$ reads

$$\bar{\lambda}^i \equiv \lambda_i^\dagger \gamma^0 = {}^t \lambda^i C, \quad (A.2)$$

where $\bar{\lambda}$ is the usual Dirac conjugate and C is the charge conjugation matrix satisfying ${}^t C = -C = C^{-1}$.

The five-dimensional $(\gamma^a)_\alpha{}^\beta$ matrices satisfy the Dirac algebra

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}.$$

The spinorial indices can be naturally raised and lowered through the use of the charge conjugation matrix $C_{\alpha\beta}$, ${}^tC = -C = C^{-1}$.

To have matrices with fixed symmetry properties we define

$$\begin{aligned} (\Gamma^{[n]})_\alpha{}^\beta &\equiv (\gamma^{[n]})_\alpha{}^\beta, & (\Gamma^{[n]})^\alpha{}_\beta &\equiv C^{\alpha\rho}(\gamma^{[n]})_\rho{}^\sigma C_{\sigma\beta}, \\ (\Gamma^{[n]})_{\alpha\beta} &\equiv (\gamma^{[n]})_\alpha{}^\sigma C_{\sigma\beta}, \text{ and } & (\Gamma^{[n]})^{\alpha\beta} &\equiv C^{\alpha\sigma}(\gamma^{[n]})_\sigma{}^\beta, \end{aligned} \quad (\text{A.3})$$

where $\gamma^{[n]}$ means the antisymmetrized product of n γ matrices with weight one: $\gamma^{[n]} = \gamma^{[a_1} \dots \gamma^{a_n]}$. Thus

$${}^t\Gamma^a = -\Gamma^a, \quad {}^t\Gamma^{ab} = \Gamma^{ab} \text{ and } {}^t\Gamma^{abc} = \Gamma^{abc}. \quad (\text{A.4})$$

We define the tangent space components of a generic p -form Φ_p according to [36]

$$\Phi_p = \frac{1}{p!} e^{\underline{a}_1} \dots e^{\underline{a}_p} \Phi_{\underline{a}_p \dots \underline{a}_1},$$

where the wedge product between forms is understood.

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